# STUDENTS' UNDERSTANDINGS OF PICTOGRAPHS AND BAR GRAPHS

**GARY ASP, JOHN DOWSEY AND HILARY HOLLINGSWORTH[1]**

**Department of Science and Mathematics Education**

**The University of Melbourne**

*This paper describes a preliminary investigation into students' understanding of pictographs and bar graphs. An interview protocol was developed and administered to 30 students (20 primary and 10 post-primary) selected at three ability levels. Students' responses were documented and categorised. They show that in relation to the set tasks, students had fairly well-developed skills in reading, interpreting and predicting from pictographs and bar graphs with increasing facility as the ability level and year level increase. Nevertheless, some interesting results related to four key dimensions of prior knowledge, missing data, scale and pattern were found, and these are reported together with some suggestions for future work in this area.*

National organisations in Australia and other countries acknowledge the increasing significance of students' abilities to construct and interpret graphs, recognising that a variety of graphical representations are used in many facets of life to disseminate information. Students at all levels of schooling are encouraged to use graphical representations as a means towards learning mathematical concepts and solving problems, as well as communicating results from investigations involving the application of mathematics to real problems. The importance of graphs and graphing in mathematics education is recognised in, for example, *A National Statement on Mathematics for Australian Schools* (1991) and *Curriculum and Evaluation Standards in School Mathematics* (1989). The increasing use of computers simplifies the task of generating graphical displays and this is likely to lead to even greater reliance upon graphs for analysing information and communicating results. Thus it is imperative that students complete compulsory schooling with a clear understanding of the essential aspects of the graphical representation of data.

Research into the nature of students' conceptualisations of graphs has been rich in some areas and sparse in others. Leinhardt, Zaslavsky and Stein (1990), in a major review of the research and

[1]The research reported in this paper was supported by a grant from the Institute of Education, University of Melbourne to all three authors and Lionel Periera-Mendoza, for a project entitled "Students' concepts of pictographs and bar graphs". The authors would like to thank their research assistant Jeni Wilson and the teachers and students who participated in the project.

theory related to cartesian graphs and graphing, identified a large amount of work that has been done in this area. Their paper pointed to directions for further research, including the need for empirical studies of the effects of computers. A recent trial of a computer-based unit on linear and quadratic functions and their graphs by Asp, Dowsey and Stacey (1993) found that students rapidly gained facility in using the technology but continued to experience difficulties reading and interpreting coordinate graphs.

In contrast, a 1977 review of empirical research on the presentation of quantitative data by MacDonald-Ross reported that "there are virtually no experiments on the use of graphs in education" (p 137) — that is, graphs such as pictographs, bar graphs, pie charts and histograms. Some more recent studies have measured the level of understanding of students when data is presented in such graphs. Curcio (1987) showed that graphing ability increased with grade level and that while students had few difficulties with the literal reading of graphs, they were often unsuccessful in answering questions requiring higher cognitive skills such as interpreting and predicting. She further found that misuse of prior knowledge led to errors and that students were persistent in their errors. Mellor (1991) and Pereira-Mendoza and Mellor (1992) similarly found that both using and misusing knowledge of the topic played a role in producing errors, and in addition, students had a tendency to impose patterns on data. They also suggested that students appeared to experience difficulty with scale. Such problems reinforce the work of pioneers like Huff (1954) who showed that changing the scale can dramatically change the appearance of a graph and lead to misinterpretation of the message contained in the actual data.

The significance of graphs and graphing in the curriculum together with the relative paucity of research on the presentation of quantitative data points to the need for more work in this area. In a seminar in 1993 at the University of Melbourne, Pereira-Mendoza reported on an exploratory study of students' understanding of pictographs and bar graphs and suggested a number of areas worthy of investigation. He suggested that students' abilities to read, interpret and predict from pictographs and bar graphs should be explored with respect to four key dimensions that have appeared to be significant in the few studies in this area thus far. These dimensions are prior knowledge, missing data, scale and pattern.

These considerations led the present authors to undertake a preliminary investigation into students' abilities to read, interpret and predict from pictographs and bar graphs, with particular attention to the four key dimensions of prior knowledge, missing data, scale and pattern, using carefully structured situations. This paper reports some initial results and discusses some implications for future research.

## PROCEDURE

Two Victorian government schools in the eastern suburbs of Melbourne, one primary and one post-primary, participated in the study. Altogether, 30 students were involved, ten from each of the year 4 and 6 levels at the primary school and ten from the year 8 level at the post-primary school. The ten students at each level were selected for involvement in the study by their teachers according to perceived ability levels of upper (U), middle (M) and lower (L). There were three, four and three students within these respective categories at years 4 and 8 and four, three and three at year 6.

Interview tasks and questions were developed to explore students' understandings with respect to pictographs and bar graphs in relation to the four aspects of interest, namely students' prior knowledge, missing data, scale and pattern. Each student was interviewed by a research assistant using a predetermined interview protocol developed by the research team and all interviews were videotaped. Student responses were documented and categorised to provide information on the four areas of interest.

During the interview, the student was presented with three tasks. Task 1 involved the student working in a familiar context with her own data; task 2 used the same context but data was provided by the research team in the form of a pictograph; task 3 presented the student with a new context with data provided by the research team in the form of a bar graph.

For task 1, the student was asked to open a 25 gm packet of Smarties and then sort the contents in some appropriate way. [For each interview, the packet contents were 5 blue, 2 brown, 1 green, 2 orange, 5 pink, 3 purple, 6 red and 2 yellow smarties.] The student was asked some questions related to her sorting and then requested to arrange the smarties in a graph to show the different numbers of each colour. The student was then asked a series of questions which required her to read, interpret and predict from her graph.

For task 2, the student was told that a number of packets of smarties had been opened and sorted into colour groups. The student was then presented with the pictograph developed from the data obtained. This pictograph involved scale with the picture symbol used on the graph representing three smarties. The interviewer then asked a number of questions related to the student's understanding of the graph.

Finally, for task 3, the student was shown a bar graph which displayed the usual mode of travelling to school of children from a (mythical) grade 6 class. After obtaining initial reactions, the interviewer asked a series of questions designed to test the student's ability to read, interpret and predict from the given bar graph.

The tasks together with a sample of questions from the interview protocol are shown in Table 1.

Table 1. Tasks and Sample Questions

**Task 1**

Student developed pictograph (dynamic) using 25 gm packet of Smarties.

**Sample questions**

- *How many red smarties are there in your collection?*
- *What is the difference in number between the smartie colour there is most of and the colour there is least of?*
- *I took one smartie out of the packet before you opened it. What colour do you think it is?*
- *If we put all of the smarties back in the packet and then drew one out, what colour do you think it might be?*

**Task 2**

Pictograph (static) based on 3 packets of Smarties (Scale: one symbol = 3 smarties)

**Sample questions**

- *What can you tell me about the smarties in my collection? [If scale not recognised: Do you know what this scale means? ]*
- *How many smarties in my collection altogether?*
- *How many more pink smarties are there than blue smarties?*
- *What amounts of each colour do you think you would get if you opened the same number of packets as I did?*

**Task 3**

Bar graph (static) based on (mythical) grade 6 class travel habits.

**Sample questions**

- *What can you tell me about the ways the grade 6 children get to school?*
- *Do more children travel to school by walking or by riding their bikes? How did you work that out?*
- *This grade six class has the same number of boys as girls. Suppose the boys and girls have similar patterns of travelling to school. How many girls do you think might come by car?*
- *Suppose you meet any one of the children in this grade and you ask them how they travel to school. Can you predict which way they would travel? Why (or why not)?*

## RESULTS AND DISCUSSION

For each task, results are presented under the headings of reading, interpreting and predicting. In this report, not all questions or all parts of questions are discussed and only the main findings are illustrated. Where references have been made to particular students, names have been changed and notation of the form (6, M) used to indicate, in this example, that the student is in year 6 and at the middle level of ability.

### Task 1

Of the 30 students, one year 4 student arranged the smarties into mixed colours, another sorted them into colour groups, and one of the year 6 students attempted a rather unusual form of straight line graph. The remainder constructed reasonable pictographs. The year 8 students tended to arrange the colours so that their frequency was ordered from highest to lowest (five students) or lowest to highest (two students) with the rest (three students) organising randomly by colour. The year 4 and 6 students were equally split between order according to frequency and random by colour.

#### *Reading*

All students other than two in year 6 were able to read the number of red smarties from their graph, and one of these was subsequently found to be seeing red smarties as brown[2]. In using their graph to determine the total number of smarties in their collection however, there were some interesting comparisons between year levels. Of the year 4s, eight obtained the correct total whereas only five of the year 6s and six of the year 8s were able to give the correct answer. The incorrect responses from the year 8s came from students in the lower and middle ability levels. This was actually the most difficult question of all for the year 8s. It may simply be that these students perceived the task as being too easy and therefore took less care in counting up.

#### *Interpreting*

All students were able to identify the most and least frequent colours from their graph. When it came to the difference in their numbers, four of the year 4s were unable to answer but all other students gave the correct answer. Some of the year 4s had a problem with the word 'difference' — for example, Susan (4, L) responded "They've got different numbers — this is 6 and that is 5; 6 is the highest". The question 'Are there any colours with the same number of smarties?' was better answered by the younger students almost all of whom provided both possible correct answers (seven of the year 4s and nine of the year 6s). In contrast, four of the year 8s only identified one of the possible correct answers.

---

[2] One of the current authors who also suffers from red-green colour blindness has much sympathy for this student!

***Predicting***

Responses to the question which required students to guess which colour had been removed before the packet was opened were rather interesting. Altogether 25 students (eight year 4s, eight year 6s and nine year 8s) opted for the colour green, the least frequent colour in the given collection. A typical explanation was given by Mark (6, U) who said "Green; you don't usually get one green — you usually get a few of each". One year 4 student selected red but could not explain why. Chris (6, U) selected red because "the average amount ... is more than the rest of them". In contrast, the question about what might happen if we put the smarties back in the packet and then drew one out resulted in quite a different response. There were 23 students who selected red (five year 4s, nine year 6s and nine year 8s) with explanations related to the fact that there were more reds than any other colour. Tristen (8, U) commented "There's a better chance of getting red but it'd probably be blue or pink 'cause there's more pink and blue combined than red". Four of the year 4s chose blue with general comments about how it could be anything, and the one year 6 who chose blue explained that it was her favourite colour.

**Task 2**

Most of the year 4 students had some initial difficulty with the concept of scale and it had to be explained to them. All but two of the year 6s and all of the year 8s were able to explain what was meant by the scale symbol.

***Reading***

When asked 'How many pink smarties in my collection?', some year 4 students initially responded "Five" — when reminded about the scale factor, they were usually able to give the correct answer. In other questions, it was clear that they knew how to work out the answer but sometimes they made an error when multiplying by 3. There were few errors of this type amongst the year 6 and 8 students. In particular, when working out the total number of smarties from the graph, nine of the year 6s and nine of the year 8s were able to give the correct answer of 75, whereas only two of the year 4s were correct (the Ls were unable to tackle this question at all, while the Ms and Us attempted to use the scale but made incorrect calculations).

***Interpreting***

All but one of the students who were able to read scale correctly interpreted the question about how many extra pink smarties there were than blue. All year 8 students and nine of the year 6 students were correct on this item and seven of the year 4 students were also correct.

### *Predicting*

The question that asked the students to predict the colour distribution if they opened the same number of packets caused considerable difficulty. Many asked for the question to be repeated. and it seemed that the complexity of the sentence led to a number of "Don't know" or "Unsure" responses. Altogether, four of the year 4s, six of the year 6s and four of the year 8s were able to give reasonable answers. In particular, the three Us from the year 4s and 6s and two of the Us from the year 8s responded that there would be about the same. Of interest is the fact that most of the younger students attempted to use the given data to predict what would happen whereas older students wanted to use their own theories or prior knowledge about what the contents of Smartie packets might be. Some for example referred to the information from the first task.

### Task 3

Some students used prior knowledge to help them in some questions on this task. In particular, younger students tried to use their 'world' knowledge to try to help them in prediction tasks whereas older students tried to use the information supplied by the graph. This is an interesting contrast to the finding in the previous paragraph.

### *Reading*

In response to the question 'What can you tell me about the ways the grade 6 children get to school?', types of responses varied at different year levels. Year 8 students tended to read the graph in a very broad sense — a typical response was "Most travel by car, hardly any by train". Only one year 8 used numbers in any way. In contrast, eight of the year 4s reported numerical information such as "9 by car, 7 by bike, ..." of which five gave the data in numerical order from highest to lowest. With respect to the graph itself, five students (some from each year level) made comments like "It's not ordered from largest to smallest". Some of the students used their own knowledge to make comments that went beyond the data. For example, Ben (8, M) commented "Ride bike, catch bus, go in car with mum and dad, catch train and walk — they might get the bus to the train station, get the train and then walk". Cameron (6, M) added "They're unfit". Year 6s tended to try to explain the data (rather than just reporting it) most often.

### *Interpreting*

The question 'Do more children travel to school by walking or by riding their bikes?' drew some interesting responses. Altogether, seven of the year 4s, eight of the year 6s and eight of the year 8s were able to give the correct answer (the bar graph shows four of each). None of the remaining students attempted to measure or check their (incorrect) answers. The two year 8 students who were wrong (one L, one M) thought that there was a slight difference in the height of the bars —

they did not read the frequency and did not realise that the frequency could not be between four and five. No students added comments that used prior knowledge in answering this item.

***Predicting***

Most students (six year 4s, seven year 6s and nine year 8s) answered the question 'How many girls do you think might come by car?' with "4" or "5" or "4 or 5" (the bar graph showed that nine children came by car). Most of the others tried to find about half of nine but made calculation errors. Reasons given tended to be mathematically based although a few of the younger students could not give a reason or said that they had just guessed or sometimes used their own experiences in some way. For example, Lisa (4, U) responded "4 ... I don't know ... You can't halve 9 ... Usually there's more boys in class". When it came to predicting the mode of travel of a child (from the mythical grade) that they had just met, eight year 8s, eight year 6s and six year 4s opted for the car (which was the mode from the bar graph). Explanations given included "More chance" (usually given by year 8 students) to a range of reasons beyond the bar graph given. For example, Richard (6, M) said it could either be by car or walking and continued "By seeing how they get to school ... there's stacks of cars and people walking on the footpath".

## SUMMARY AND CONCLUDING REMARKS

Some general behaviours seem to recur across the tasks. Asking "Anything else?" or "Why?" after the initial reply usually elicited further responses with younger students showing a greater tendency than older students to use prior ('world') knowledge and a greater likelihood of sticking to incorrect conclusions despite what the data indicated. Problems with scale appeared in two ways with younger students. Some students were simply unfamiliar with the idea and needed an explanation of it — this did not seem to be a problem at all with older students. The other problem with scale appeared in task 2 where there was no symbol in the green category, indicating zero green smarties in the collection. The presence of the 'green' category and scale in the graph led some students to think that there could be as many as two or three green smarties. Finally, pattern cropped up in two contexts. Many students created their smartie pictograph and then re-arranged it so that the colours were sequenced from lowest to highest or vice versa, as if this was an essential component of such a graph. In the case of the bar graph, a number of students, particularly year 4s and 6s, commented on the graph not being drawn in frequency order — again, it seems that some students believe that graphs have to show a more definite pattern and they were unhappy about the random arrangement of the categories.

The results so far point to the need for further work on the effects of prior knowledge, scale, and pattern on student pictograph and bar graph comprehension (reading, interpreting and predicting).

Gender could be another focus for any future research, for although this preliminary investigation did not initially specify gender as an issue of concern, it did appear that there are some differences between females and males in their responses to a number of questions.

## REFERENCES

Asp, G., Dowsey, J. and Stacey, K. (1993). Linear and quadratic graphs with the aid of technology. In *Proceedings of the Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia.* MERGA, Kelvin Grove.

Australian Education Council (1991). *A National Statement on Mathematics for Australian Schools.* Canberra: AEC.

Curcio, F. R. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education,* **18** (5), 382-393.

Huff, D. (1954). *How to lie with statistics.* New York: Neston.

Leinhardt, G., Zaslavsky, O. and Stein, M. K. (1990). Functions, graphs, and graphing: tasks, learning, and teaching. *Review of Educational Research,* **60** (1) 1-64.

MacDonald-Ross, M. (1977). How numbers are shown. *Audio Visual Communications Review,* **25** (4), 359-409.

Mellor, J. (1991). A descriptive study of grade four and grade six students' understanding of bar graphs. Unpublished Masters Thesis, Memorial University of Newfoundland.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards in School Mathematics.* Reston, Va: NCTM.

Pereira-Mendoza, L. and Mellor, J. (1992). Students' concepts of pictographs and bar graphs — exploratory studies. Paper presented at the Annual Meeting of the Council for Diagnostic and Prescriptive Mathematics. Princeton, NJ.